

Hard approximation in two-particle hadronic decays of B_c at large recoils

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The two-particle decays of $B_c^+ \rightarrow \psi\pi^+(\rho^+)$ and $B_c^+ \rightarrow \eta_c\pi^+(\rho^+)$ are considered in a way taking into account a soft binding of quarks in the heavy quarkonia and a hard gluon exchange between the constituents at large recoil momenta of $\psi(\eta_c)$. An approximate double enhancement of the amplitudes is found because of the nonspectator t -channel contribution.

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I. INTRODUCTION

The QCD dynamics plays a significant role in an extraction of the electroweak theory parameters in the heavy quark sector. One of the systems allowing one to perform an exact numerical study of the heavy quark interactions, is the $(\bar{b}c)$ system, the heavy quarkonium with the mixed flavor. At present, the experimental search for the B_c meson, the basic pseudoscalar $1S$ -state of the $(\bar{b}c)$ system, is carried out at CDF [1] and ALEPH [2].

General properties of the B_c meson family can be quite reliably predicted in the theoretical investigations allowing one to make an objective experimental search for the B_c observation (see the review on the B_c physics in [3]). The spectroscopic characteristics of $(\bar{b}c)$ family can be calculated in the framework of phenomenological nonrelativistic potential models [4,5] and their relativistic modifications [6]. The strong and electromagnetic interactions conserving the flavor, do not give the annihilation modes of the $(\bar{b}c)$ state decays. Therefore, the excited levels radiatively transform into the lowest longliving pseudoscalar B_c^+ state decaying due to the weak interaction. The mass of this state, $M(B_c^+) = 6.25 \pm 0.03$ GeV, and its leptonic constant, $f_{B_c} = 385 \pm 25$ MeV, can be predicted in the framework of potential models [4,5], QCD sum rules [7–9] and in the lattice computations [10]. The life time, $\tau(B_c) = 0.55 \pm 0.15$ ps was estimated in several papers, where one took into account corrections caused by the quark binding inside the heavy quarkonium in two ways, the phenomenological one [11] as well as in the operator product expansion for the weak currents of decays of the heavy quarks composing the B_c meson [12].

From the viewpoint of the experimental selection of the B_c meson signal in a hadronic background, the preferable modes for the B_c observation are those, wherein the final state contains the ψ particle, which can be reliably identified in the leptonic decay, $\psi \rightarrow l^+l^-$. As \bar{c} -quark produced in the $\bar{b} \rightarrow \bar{c}W^{*+}$ transition, can bind the spectator c -quark of the B_c meson with a high probability into the $(\bar{c}c)$ meson, the relative yield of ψ particles in the B_c decays should be enhanced in comparison with the branching ratio of the $B_{u,d}$ meson decay modes with ψ in the final state. Indeed, under the obtained theoretical estimates in the framework of phenomenological models of the meson, one should expect $\text{BR}(B_c^+ \rightarrow \psi X) \sim 17\%$, which is much greater than $\text{BR}(B_{u,d} \rightarrow \psi X) \sim 1\%$.

As for the semileptonic decay mode of $B_c^+ \rightarrow \psi l^+ \nu$, estimates of its width calculated within the potential models [11,13] and in the QCD sum rules [9], point out the essential discrepancy between results obtained in these two approaches (the QCD sum rule estimate of the $B_c^+ \rightarrow \psi l^+ \nu$ decay width is one order of magnitude less than values given by the different models of heavy quarkonia). As was shown in [14], this deviation can be removed, if one takes into account the Coulomb corrections to the vertices of the meson quantum-number currents in the framework of QCD sum rules.

The semileptonic mode of B_c decay is suitable for the reliable experimental identification of the B_c meson at a rather large statistics of events with B_c [15]. However, at the current experiments in e^+e^- -annihilation and hadron-hadron collisions, one has to expect the B_c production rate, which evidently is not sufficient to identify B_c in the semileptonic mode [16,17]. Therefore, the two-particle decay of $B_c^+ \rightarrow \psi\pi^+$ allowing one to find B_c practically over a single event, is of the greatest interest in the experimental search for B_c . The estimate of its width calculated in the potential models, gives the branching fraction

$$\text{BR}^{\text{PM}}(B_c^+ \rightarrow \psi\pi^+) \approx 0.2\%.$$

However, in the semileptonic B_c decay the region of low momenta for the ψ particle recoil dominates, and this allows one to apply the approximate spin symmetry for the heavy $(\bar{b}c)$ and $(\bar{c}c)$ quarkonia [18] and reliably to use the

way of the transition form-factor calculation under the overlapping of the quarkonium wave functions. In contrast to the above transition, the two-particle modes of hadronic decays of $B_c^+ \rightarrow \psi\pi^+(\rho^+)$ and $B_c^+ \rightarrow \eta_c\pi^+(\rho^+)$ require a special consideration. This is related with the fact that at large momenta of the recoil quark in the $\bar{b} \rightarrow \bar{c}\pi^+$ transition, the \bar{c} -antiquark has to exchange by a hard gluon with the charmed c -quark being in the initial state, to form the bound $\psi(nS)$ or $\eta_c(nS)$ state in the region of low invariant masses of the $(\bar{c}c)$ pair due to nonperturbative soft interactions of QCD. Thus, the feature of the two-particle hadronic B_c decays is determined by the fact that in the \bar{b} -quark decays, the spectator quark is also heavy and, hence, at large energy release, the description of exclusive production of the $(\bar{c}c)$ quarkonium in the final state can not be performed in the framework of the spectator approach, where the quark-spectator determines only the amplitude of a soft forming of the bound state, so that the process of the hard weak decay can be factorized and it does not depend on the spectator. In the decays under consideration, this spectator picture is not valid. So, one can use the Brodsky-Lepage hard scattering formalism [19].

In this paper, we consider the exclusive $B_c^+ \rightarrow \psi\pi^+(\rho^+)$ and $B_c^+ \rightarrow \eta_c\pi^+(\rho^+)$ decays in the framework of the hard approximation at large recoils with taking into account the gluon exchange to the c -quark in the initial state. In contrast to the spectator approach, the hard t -channel exchange results in the approximate double enhancement of the decay amplitudes, as it was recently found for $B_c^+ \rightarrow \psi\pi^+$ [20].

In Section II we derive expressions for the amplitudes and widths of the $B_c^+ \rightarrow \psi\pi^+(\rho^+)$ and $B_c^+ \rightarrow \eta_c\pi^+(\rho^+)$ decays and compare them with the spectator formulae for the $\bar{b} \rightarrow \bar{c}\pi^+(\rho^+)$ transitions. Numerical estimates of the decay widths are given in Section III, where theoretical uncertainties of the values are discussed. The obtained results are summarized in the Conclusion.

II. CALCULATION OF TWO-PARTICLE WIDTHS OF B_C

In the framework of the nonrelativistic formalism for the heavy quark binding into the S -wave quarkonium, we assume that the momentum of the quark, composing the meson, is equal to $p_Q^\mu = m_Q v^\mu$, where v_μ is the four-velocity of quarkonium, so that the quarks inside the meson move with the same four-velocity v . Moreover, the quark-meson vertex with nontruncated quark lines corresponds to the spinor matrix

$$\Gamma_V = \hat{\epsilon} \frac{1 + \hat{v}}{2} \frac{\tilde{f} M_{nS}}{2\sqrt{3}},$$

for the vector quarkonium with ϵ_μ , being the polarization vector, and

$$\Gamma_P = \gamma_5 \frac{1 + \hat{v}}{2} \frac{\tilde{f} M_{nS}}{2\sqrt{3}},$$

for the pseudoscalar quarkonium. Here M_{nS} is the nS -level mass and \tilde{f} is related with the value of configuration wave function at the origin

$$\tilde{f} = \sqrt{\frac{12}{M_{nS}}} |\Psi_{nS}(0)|.$$

The \tilde{f} quantity can be related with the leptonic constants of states

$$\begin{aligned} \langle 0 | J_\mu(0) | V \rangle &= i f_V M_V \epsilon_\mu, \\ \langle 0 | J_{5\mu}(0) | P \rangle &= i f_P p_\mu, \end{aligned}$$

where $J_\mu(x)$ and $J_{5\mu}(x)$ are the vector and axial-vector currents of the constituent quarks. Then the allowance for the hard gluon corrections in the first order over α_s [7,21–23] results in

$$\tilde{f} = f_V \left[1 - \frac{\alpha_s^H}{\pi} \left(\frac{m_2 - m_1}{m_2 + m_1} \ln \frac{m_2}{m_1} - \frac{8}{3} \right) \right], \quad (1)$$

$$\tilde{f} = f_P \left[1 - \frac{\alpha_s^H}{\pi} \left(\frac{m_2 - m_1}{m_2 + m_1} \ln \frac{m_2}{m_1} - 2 \right) \right], \quad (2)$$

where $m_{1,2}$ are the masses of quarks composing the quarkonium. For the vector currents of quarks with equal masses, the BLM procedure of the scale fixing in the "running" coupling constant of QCD [24] gives [23]

$$\alpha_s^H = \alpha_s^{\overline{\text{MS}}}(e^{-11/12} m_Q^2).$$

For the quarkonium with $m_1 \neq m_2$, we assume

$$\alpha_s^H = \alpha_s^{\overline{\text{MS}}}(e^{-11/12} m_1 m_2) .$$

Note, in the given estimates one considers the hard gluon corrections to the quark-antiquark annihilation currents. The corresponding factors are known exactly, and that is surprisingly, they can be obtained by the symbolic substitutions $m_1 \rightarrow -m_1$, $V \leftrightarrow P$ from the exact expressions for the hard gluon factors of the quark-to-quark transition currents [21], considered in HQET [25], at the prescription of the absolute value for the logarithm argument. However, these substitutions do not lead to valid evaluations of the BLM scales determining α_s^H . The corresponding BLM scales in HQET have been calculated by M. Neubert [26], and they do not give the exactly known result for the quark-antiquark annihilation vector current [23].

Further, the factor of the colour wave function $\delta^{ij}/\sqrt{3}$ stands in the quark-meson vertex.

The π meson current corresponds to the axial-vector current of weak transition $A^\mu = f_\pi p_\pi^\mu$. So, the given factorization neglects possible final state interactions, which really seem to be small (see discussion in [27]).

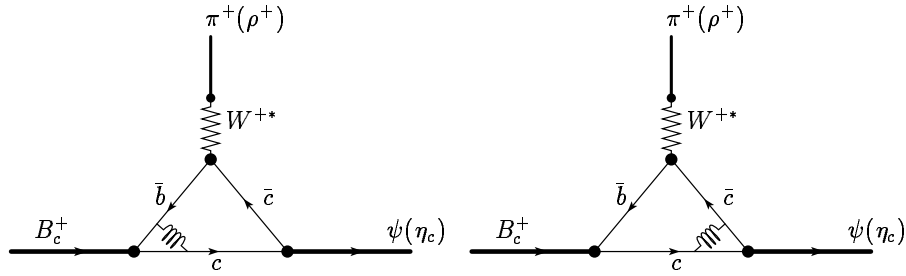


Figure 1: Diagrams of the $B_c^+ \rightarrow \psi(\eta_c)\pi^+(\rho^+)$ decays with the hard gluon exchange between the constituent quarks.

Then the matrix elements of $B_c^+ \rightarrow \psi\pi^+$ and $B_c^+ \rightarrow \eta_c\pi^+$ decays calculated according to the diagrams in Fig. 1, take the form

$$T(B_c^+ \rightarrow \psi\pi^+) = \frac{G_F}{\sqrt{2}} V_{bc} \frac{32\pi\alpha_s}{9} f_\pi \tilde{f}_{B_c} \tilde{f}_\psi \frac{M^2}{m_\psi^2(y-1)^2} (\epsilon \cdot v) a_1 , \quad (3)$$

$$T(B_c^+ \rightarrow \eta_c\pi^+) = \frac{G_F}{\sqrt{2}} V_{bc} \frac{8\pi\alpha_s}{9} f_\pi \tilde{f}_{B_c} \tilde{f}_{\eta_c} \frac{(M^2 - m_{\eta_c}^2)(3M^2 - 2Mm_{\eta_c} + m_{\eta_c}^2)}{Mm_{\eta_c}^3(y-1)^2} a_1 , \quad (4)$$

where v is the four-velocity of B_c meson, M is its mass, ϵ is the polarization vector of ψ particle, m_ψ is its mass, $y = v \cdot v_\psi$ is the product of the B_c and ψ four-velocities,

$$y = \frac{M^2 + m_\psi^2}{2Mm_\psi} .$$

In (4) the notations for the η_c state are analogous to the described ones. The a_1 factor is caused by the hard gluon corrections to the effective four-fermion weak interactions of quarks. a_1 is evaluated for free quarks, so that $a_1 = 1.22 \pm 0.04$ [3]. In present calculations, this correction is at the same level as the hard corrections to the decay constants as well as the gluon propagator. The value of the QCD coupling constant is determined by the gluon virtuality, $k_g^2 = -m_{\psi,\eta_c}^2(y-1)/2$, and it will be discussed in next Section.

The corresponding virtualities of \bar{c} and \bar{b} quarks, interacting with the hard gluon, are equal to

$$\begin{aligned} k_c^2 - m_c^2 &= 2k_g^2 , \\ k_b^2 - m_b^2 &= 2k_g^2 M/m_{\psi,\eta_c} . \end{aligned}$$

Note, that numerically the virtual \bar{c} -quark in the second diagram in Fig. 1 is in the t -channel, since its four-momentum squared has a negative value. Therefore, one can see, that the corresponding contribution into the $B_c^+ \rightarrow \psi(\eta_c)\pi^+$ decay is definitely nonspectator, and the considered process is certainly hard.

From Eqs.(3,4) one gets the expressions for the total widths of the $B_c^+ \rightarrow \psi\pi^+$ and $B_c^+ \rightarrow \eta_c\pi^+$ decays

$$\Gamma(B_c^+ \rightarrow \psi\pi^+) = G_F^2 |V_{bc}|^2 \frac{128\pi\alpha_s^2}{81} f_\pi^2 \tilde{f}_{B_c}^2 \tilde{f}_\psi^2 \left(\frac{M + m_\psi}{M - m_\psi} \right)^3 \frac{M^3}{(M - m_\psi)^2 m_\psi^2} a_1^2, \quad (5)$$

$$\Gamma(B_c^+ \rightarrow \eta_c\pi^+) = \Gamma(B_c^+ \rightarrow \psi\pi^+) \cdot \frac{(3M^2 - 2Mm_{\eta_c} + m_{\eta_c}^2)^2}{4M^4}. \quad (6)$$

As for the analogous two-particle B_c^+ decays with ρ^+ in the final state, one uses the approximate factorization of the transition current of the virtual W^{*+} boson into π^+ or ρ^+ , and one finds that the only difference between the squares of amplitudes for the pseudoscalar and vector states of the light quark systems, is the substitution of the quantity $f_\rho^2 m_\rho^2 (-g_{\mu\nu} + p_\mu p_\nu / m_\rho^2)$ instead of the π^+ meson current tensor $f_\pi^2 p_\mu p_\nu$. Then one can easily observe that after the summation over the ρ meson polarizations, the squares of the matrix elements coincide up to the factor, so

$$\frac{\Gamma(B_c^+ \rightarrow \psi\rho^+)}{\Gamma(B_c^+ \rightarrow \psi\pi^+)} \approx \frac{\Gamma(B_c^+ \rightarrow \eta_c\rho^+)}{\Gamma(B_c^+ \rightarrow \eta_c\pi^+)} \approx \frac{f_\rho^2}{f_\pi^2}, \quad (7)$$

in the leading order over the small parameters, m_ρ^2/m_c^2 , m_ρ^2/m_b^2 .

Write down the expressions for the spectator decays of b -quark

$$\Gamma(\bar{b} \rightarrow \bar{c}\pi^+) = G_F^2 |V_{bc}|^2 \frac{m_b^3 f_\pi^2}{16\pi} \left(1 - \frac{m_c^2}{m_b^2} \right)^3 a_1^2, \quad (8)$$

$$\Gamma(\bar{b} \rightarrow \bar{c}\rho^+) = \Gamma(\bar{b} \rightarrow \bar{c}\pi^+) \frac{f_\rho^2}{f_\pi^2} \left\{ 1 + \frac{m_\rho^2(m_b^2 + m_c^2)}{(m_b^2 - m_c^2)^2} \right\}. \quad (9)$$

From Eqs.(8,9) one can see that for the spectator decays the relation between the yields of ρ^+ and π^+ mesons

$$\frac{\Gamma(\bar{b} \rightarrow \bar{c}\rho^+)}{\Gamma(\bar{b} \rightarrow \bar{c}\pi^+)} \approx \frac{f_\rho^2}{f_\pi^2}$$

is valid in the leading approximation over the square of ratios of the ρ meson mass over the heavy quark masses, as it takes place for the transitions between the mesons (see (7)). In the spectator decays the accuracy of the leading approximation used, is about 4 %, that also points out the magnitude of the correction terms to relation (7) for the mesons.

The breaking of the spectator picture at large recoils in nonhadronic decays of B_c^+ was also recently considered in [28], where one studied the $B_c^+ \rightarrow D_s^+ \gamma$ mode due to the flavor-changing neutral current of $\bar{b} \rightarrow \bar{s} \gamma$. The discussion of the heavy quark symmetry at large recoils in the heavy-light meson transitions was given in [29], where the peaking approximation was introduced. This approximation corresponds to the quark-meson vertices used in this paper.

III. NUMERICAL ESTIMATES

The accuracy of the given calculations is basically restricted by the uncertainty in the choice of the QCD coupling constant value. In Eqs.(3,4) α_s can be evaluated at the scale typical for the charm quark physics $\alpha_s \approx 0.30$. The higher order corrections are beyond the scope of this paper. Nevertheless, to evaluate the possible value of these corrections, one can use the BLM procedure including the light quark loops in the virtual gluon propagator. So, α_s is given at the scale of the gluon virtuality

$$k_g^2 = -m_{\psi, \eta_c}^2 (y - 1)/2 \approx -1.2 \text{ GeV}^2, \quad (10)$$

so that

$$\alpha_s = \alpha_s^{\overline{\text{MS}}}(-e^{-5/3} k_g^2).$$

As one can see from Eq.(10), the virtuality of hard gluon is comparable with the square of charm quark mass, and it indicates the applicability of the hard process factorization. Moreover, the scheme-independent value of the α_s argument $e^{-C_{\text{scheme}} k^2 / \Lambda_{\text{scheme}}^2}$ is quite large, and it is close to 15. Numerically, the BLM fixing of the QCD coupling constant gives $\alpha_s^{\text{BLM}} \approx 0.57$. In the above procedure the gluon virtualities are taken into account. However, the

quark virtualities can be essential, too. Indeed, the charmed quark virtuality is valued in the intermediate region, where it is less than the b -quark one and greater than the gluon virtuality. So, in the following estimates we will use the value

$$\alpha_s = \alpha_s^{\overline{\text{MS}}}(m_c^2 - k_c^2) \approx \alpha_s^{\overline{\text{MS}}}(-2k_g^2) = 0.33 \pm 0.06 .$$

The uncertainty in the α_s value appropriate for the given process indicates a possible large role of higher order corrections.

In numerical estimates we suppose [30]

$$|V_{bc}| = 0.041 \pm 0.003 ,$$

and we use the one-loop expression for the α_s evolution

$$\alpha_s(m^2) = \frac{4\pi}{\beta_0(n_f) \ln(m^2/\Lambda_{(n_f)}^2)} ,$$

where $\beta_0(n_f) = 11 - 2n_f/3$, n_f is the number of quark flavors with $m_{n_f} < m$,

$$\Lambda_{(n_f)} = \Lambda_{(n_f+1)} \left(\frac{m_{n_f+1}}{\Lambda_{(n_f+1)}} \right)^{\frac{2}{3\beta_0(n_f)}} .$$

Using $\alpha_s^{\overline{\text{MS}}}(m_Z^2) = 0.117 \pm 0.005$ [31], one finds that $\Lambda_{(5)} = 85 \pm 25$ MeV and $\Lambda_{(3)} = 140 \pm 40$ MeV. One estimates $\alpha_s^{\overline{\text{MS}}}(m_b^2) = 0.20 \pm 0.02$, that is quite reasonable.

The f_{B_c} constant was estimated in the framework of the QCD sum rules [7–9]

$$f_{B_c} = 385 \pm 25 \text{ MeV} ,$$

and it is in a good agreement with the scaling relation for the leptonic constants of $1S$ heavy quarkonia [7]

$$\frac{f^2}{M} \left(\frac{M}{\mu_{12}} \right)^2 = \text{const.} , \quad \mu_{12} = \frac{m_1 m_2}{m_1 + m_2} .$$

Then, the account for the hard gluon corrections gives

$$\tilde{f}_\psi = \tilde{f}_{\eta_c} = 542 \pm 50 \text{ MeV} , \tag{11}$$

$$\tilde{f}_{B_c} = 440 \pm 40 \text{ MeV} . \tag{12}$$

To calculate the branching ratios we evaluate the total B_c meson width according to the formula by [3,11]

$$\Gamma(B_c) \approx \Gamma(B) + (0.6 \pm 0.1)\Gamma(D^+) + \Gamma(ann.) ,$$

where $\Gamma(B)$ is the contribution of \bar{b} -quark decays with the spectator c -quark, $\Gamma(D^+)$ determines the contribution of c -quark decays with the spectator \bar{b} -quark and with the account for the phase space reduction, because of the c -quark binding inside B_c (i.e. one takes into account the deviation from the exact spectator consideration), and $\Gamma(ann.)$ is the contribution of annihilation channels depending on $|V_{bc}|$ and f_{B_c} . Then

$$\Gamma(B_c) = (1.2 \pm 0.2) \cdot 10^{-3} \text{ eV} = \frac{1}{0.55 \pm 0.15 \text{ ps}} .$$

The recent estimates of the $\Gamma(B_c)$ total width calculated within the operator product expansion approach and including also the bound quark effects, annihilation channels as well as the Pauli interference in the final state of B_c decay, is in a good agreement with the given value [12].

Supposing $a_1 = 1.22 \pm 0.04$ [3], $f_\rho = 220$ MeV, one finally finds

$$\Gamma(B_c^+ \rightarrow \psi \pi^+) = (9.2 \pm 2.3) \cdot 10^{-6} \text{ eV} = \frac{1}{69 \pm 17 \text{ ps}} , \tag{13}$$

$$\Gamma(B_c^+ \rightarrow \psi \rho^+) = (24 \pm 6) \cdot 10^{-6} \text{ eV} = \frac{1}{22 \pm 5 \text{ ps}} , \tag{14}$$

and

$$\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi\pi^+) = 0.77 \pm 0.19\% , \quad \text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c\pi^+) = 1.00 \pm 0.25\% , \quad (15)$$

$$\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi\rho^+) = 2.25 \pm 0.56\% , \quad \text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c\rho^+) = 2.78 \pm 0.70\% . \quad (16)$$

Further, the purely spectator decays of $\bar{b} \rightarrow \bar{c}\pi^+(\rho^+)$ have the following branching fractions with respect to the total B_c^+ width

$$\text{BR}^{\text{spec}}(\bar{b} \rightarrow \bar{c}\pi^+) \approx 0.64\% , \quad (17)$$

$$\text{BR}^{\text{spec}}(\bar{b} \rightarrow \bar{c}\rho^+) \approx 1.8\% , \quad (18)$$

The matrix element, corresponding to the first diagram in Fig. 1, is approximately equal to the matrix element, following from the second diagram and, hence, estimates (15,16) are enhanced by a factor of four due to the t -exchange nonspectator contribution.

As for the nS -excitation yields of the $(\bar{c}c)$ quarkonium in the B_c^+ decays, we note that the corresponding branching fractions are determined by the rescaling of the leptonic constants and phase spaces. So,

$$\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi(nS)\pi^+(\rho^+)) = \text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi\pi^+(\rho^+)) \frac{f_{nS}^2}{f_\psi^2} \frac{M^2 - m_{nS}^2}{M^2 - m_\psi^2} . \quad (19)$$

The experimental values of leptonic constants are in a good agreement with the scaling expression [7]

$$\frac{f_{nS}^2}{f_\psi^2} = \frac{1}{n} \frac{m_\psi}{m_{nS}} . \quad (20)$$

So, neglecting the differences in the masses of $\psi(nS)$ and $\eta_c(nS)$ states, one gets

$$\frac{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi(2S)\pi^+)}{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi\pi^+)} = \frac{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c(2S)\pi^+)}{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c\pi^+)} \approx 0.36 , \quad (21)$$

for instance. The same values for the $\psi(nS)\rho^+$ state yields can be rewritten down.

IV. CONCLUSION

In this paper we have shown that in the $B_c^+ \rightarrow \psi\pi^+(\rho^+)$ and $B_c^+ \rightarrow \eta_c\pi^+(\rho^+)$ decays the large momentum of the recoil ψ or η_c particle leads to the fact that the formalism of the weak transition form-factor calculation, based on the overlapping of the nonrelativistic wave functions for the heavy quarkonia, is not valid. The hard gluon exchange with the spectator quark results in the large virtuality of heavy quark in the weak transition current. The amplitude of the weak decay with the hard exchange by gluon can be calculated in the framework of QCD perturbation theory and this amplitude can be factorized from the amplitude of soft binding of heavy quarks in the quarkonium. The calculations with the account for this hard-soft factorization result in

$$\begin{aligned} \text{BR}(B_c^+ \rightarrow \psi\pi^+) &= 0.77 \pm 0.19\% , \\ \text{BR}(B_c^+ \rightarrow \psi\rho^+) &\approx 2.78 \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+) , \\ \text{BR}(B_c^+ \rightarrow \eta_c\pi^+(\rho^+)) &\approx 1.28 \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+(\rho^+)) , \end{aligned}$$

where the accuracy is basically restricted by uncertainties in the evolution scale of the "running" QCD coupling constant, in the c -quark mass, and the total B_c width. The given estimate for the branching ratio of the $B_c^+ \rightarrow \psi\pi^+$ decay mode is significantly larger than the extrapolation results by the potential models. This value strongly enhance the probability of B_c observation in the current Tevatron and LEP experiments with vertex detectors.

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